

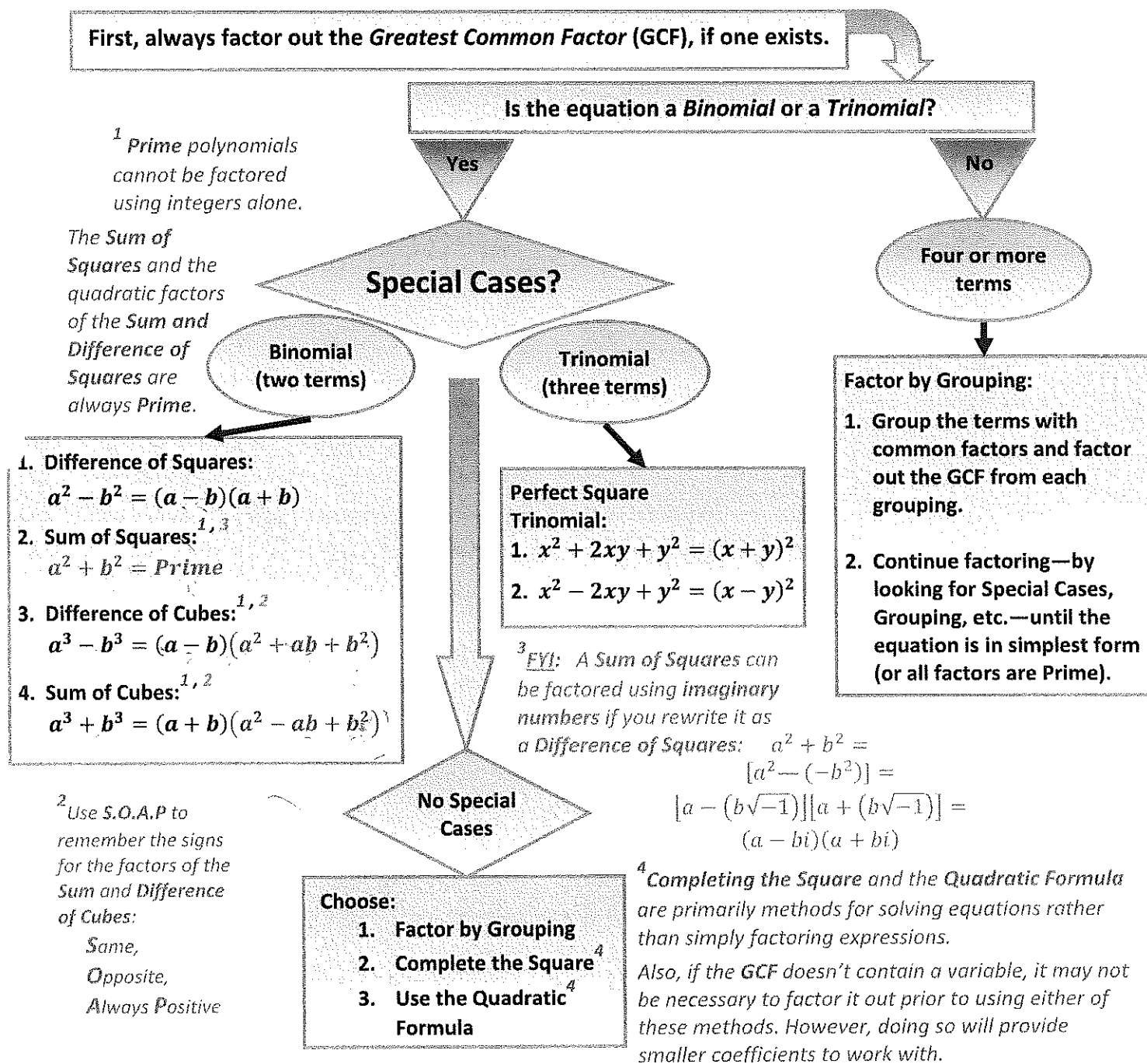
# Factoring Polynomials

**Factoring** a polynomial is the process of writing it as the product of two or more polynomial factors.

Example:  $7x^2 + 35x + 42 = 7(x + 2)(x + 3)$ —one monomial factor (7) and two binomial factors ( $x + 2$ ) and ( $x + 3$ )

Set the factors of a polynomial equation (as opposed to an expression) equal to zero in order to solve for a variable: Example: To solve  $7x^2 + 35x + 42 = 0 \rightarrow x + 2 = 0, x = -2$ ; and  $x + 3 = 0, x = -3$

The flowchart below illustrates a sequence of steps for factoring polynomials.



This process is applied in the following examples

Factoring steps and most examples are adapted from Professor Elias Juridini, Lamar State College-Orange.



## Factoring Trinomials

### Learning Objective(s)

- Factor trinomials with a leading coefficient of 1.
- Factor trinomials with a common factor.
- Factor trinomials with a leading coefficient other than 1.

### Introduction

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A **polynomial** with three terms is called a **trinomial**. Trinomials often (but not always!) have the form  $x^2 + bx + c$ . At first glance, it may seem difficult to factor trinomials, but you can take advantage of some interesting mathematical patterns to factor even the most difficult-looking trinomials.

So, how do you get from  $6x^2 + 2x - 20$  to  $(2x + 4)(3x - 5)$ ? Let's take a look.

### Factoring Trinomials: $x^2 + bx + c$

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Trinomials in the form  $x^2 + bx + c$  can often be factored as the product of two **binomials**. Remember that a binomial is simply a two-term polynomial. Let's start by reviewing what happens when two binomials, such as  $(x + 2)$  and  $(x + 5)$ , are multiplied.

Example	
<b>Problem</b>	Multiply $(x + 2)(x + 5)$ .
	$(x + 2)(x + 5)$ Use the FOIL method to multiply binomials. $x^2 + 5x + 2x + 10$ Then combine like terms $2x$ and $5x$ .
<b>Answer</b>	$x^2 + 7x + 10$

Factoring is the reverse of multiplying. So let's go in reverse and factor the trinomial  $x^2 + 7x + 10$ . The individual terms  $x^2$ ,  $7x$ , and  $10$  share no common factors. So look at rewriting  $x^2 + 7x + 10$  as  $x^2 + 5x + 2x + 10$ .

And, you can group pairs of factors:  $(x^2 + 5x) + (2x + 10)$   
 Factor each pair:  $x(x + 5) + 2(x + 5)$   
 Then factor out the common factor  $x + 5$ :  $(x + 5)(x + 2)$

Here is the same problem done in the form of an example:

Example

<b>Problem</b>	<b>Factor <math>x^2 + 7x + 10</math>.</b>
$x^2 + 5x + 2x + 10$	Rewrite the middle term $7x$ as $5x + 2x$ .
$x(x + 5) + 2(x + 5)$	Group the pairs and factor out the common factor $x$ from the first pair and $2$ from the second pair.
$(x + 5)(x + 2)$	Factor out the common factor $(x + 5)$ .
<i>Answer</i>	$(x + 5)(x + 2)$

How do you know how to rewrite the middle term? Unfortunately, you can't rewrite it just any way. If you rewrite  $7x$  as  $6x + x$ , this method won't work. Fortunately, there's a rule for that.

### Factoring Trinomials in the form $x^2 + bx + c$

To factor a trinomial in the form  $x^2 + bx + c$ , find two integers,  $r$  and  $s$ , whose product is  $c$  and whose sum is  $b$ .

Rewrite the trinomial as  $x^2 + rx + sx + c$  and then use grouping and the distributive property to factor the polynomial. The resulting factors will be  $(x + r)$  and  $(x + s)$ .

For example, to factor  $x^2 + 7x + 10$ , you are looking for two numbers whose sum is  $7$  (the coefficient of the middle term) and whose product is  $10$  (the last term).

Look at factor pairs of  $10$ :  $1$  and  $10$ ,  $2$  and  $5$ . Do either of these pairs have a sum of  $7$ ? Yes,  $2$  and  $5$ . So you can rewrite  $7x$  as  $2x + 5x$ , and continue factoring as in the example above. Note that you can also rewrite  $7x$  as  $5x + 2x$ . Both will work.

Let's factor the trinomial  $x^2 + 5x + 6$ . In this polynomial, the  $b$  part of the middle term is  $5$  and the  $c$  term is  $6$ . A chart will help us organize possibilities. On the left, list all possible factors of the  $c$  term,  $6$ ; on the right you'll find the sums.

<b>Factors whose product is 6</b>	<b>Sum of the factors</b>
$1 \cdot 6 = 6$	$1 + 6 = 7$
$2 \cdot 3 = 6$	$2 + 3 = 5$

There are only two possible factor combinations,  $1$  and  $6$ , and  $2$  and  $3$ . You can see that  $2 + 3 = 5$ . So  $2x + 3x = 5x$ , giving us the correct middle term.

### Example

<b>Problem</b>	Factor $x^2 + 5x + 6$ .
	$x^2 + 2x + 3x + 6$ Use values from the chart above. Replace $5x$ with $2x + 3x$ . $(x^2 + 2x) + (3x + 6)$ Group the pairs of terms. $x(x + 2) + (3x + 6)$ Factor $x$ out of the first pair of terms. $x(x + 2) + 3(x + 2)$ Factor 3 out of the second pair of terms. $(x + 2)(x + 3)$ Factor out $(x + 2)$ .
<b>Answer</b>	$(x + 2)(x + 3)$

Note that if you wrote  $x^2 + 5x + 6$  as  $x^2 + 3x + 2x + 6$  and grouped the pairs as  $(x^2 + 3x) + (2x + 6)$ ; then factored,  $x(x + 3) + 2(x + 3)$ , and factored out  $x + 3$ , the answer would be  $(x + 3)(x + 2)$ . Since multiplication is commutative, the order of the factors does not matter. So this answer is correct as well; they are equivalent answers.

Finally, let's take a look at the trinomial  $x^2 + x - 12$ . In this trinomial, the  $c$  term is  $-12$ . So look at all of the combinations of factors whose product is  $-12$ . Then see which of these combinations will give you the correct middle term, where  $b$  is 1.

<b>Factors whose product is <math>-12</math></b>	<b>Sum of the factors</b>
$1 \cdot -12 = -12$	$1 + -12 = -11$
$2 \cdot -6 = -12$	$2 + -6 = -4$
$3 \cdot -4 = -12$	$3 + -4 = -1$
<b><math>4 \cdot -3 = -12</math></b>	<b><math>4 + -3 = 1</math></b>
$6 \cdot -2 = -12$	$6 + -2 = 4$
$12 \cdot -1 = -12$	$12 + -1 = 11$

There is only one combination where the product is  $-12$  and the sum is 1, and that is when  $r = 4$ , and  $s = -3$ . Let's use these to factor our original trinomial.

<b>Example</b>	
<b>Problem</b>	Factor $x^2 + x - 12$
	$x^2 + 4x + -3x - 12$ Rewrite the trinomial using the values from the chart above. Use values $r = 4$ and $s = -3$ . $(x^2 + 4x) + (-3x - 12)$ Group pairs of terms.

$x(x + 4) + (-3x - 12)$  Factor  $x$  out of the first group.

$x(x + 4) - 3(x + 4)$  Factor  $-3$  out of the second group.

$(x + 4)(x - 3)$  Factor out  $(x + 4)$ .

*Answer*

$(x + 4)(x - 3)$

In the above example, you could also rewrite  $x^2 + x - 12$  as  $x^2 - 3x + 4x - 12$  first. Then factor  $x(x - 3) + 4(x - 3)$ , and factor out  $(x - 3)$  getting  $(x - 3)(x + 4)$ . Since multiplication is commutative, this is the same answer.

### Factoring Tips

Factoring trinomials is a matter of practice and patience. Sometimes, the appropriate number combinations will just pop out and seem so obvious! Other times, despite trying many possibilities, the correct combinations are hard to find. And, there are times when the trinomial cannot be factored.

While there is no foolproof way to find the right combination on the first guess, there are some tips that can ease the way.

#### Tips for Finding Values that Work

When factoring a trinomial in the form  $x^2 + bx + c$ , consider the following tips.

Look at the  $c$  term first.

- If the  $c$  term is a positive number, then the factors of  $c$  will both be positive or both be negative. In other words,  $r$  and  $s$  will have the same sign.
- If the  $c$  term is a negative number, then one factor of  $c$  will be positive, and one factor of  $c$  will be negative. Either  $r$  or  $s$  will be negative, but not both.

Look at the  $b$  term second.

- If the  $c$  term is positive and the  $b$  term is positive, then both  $r$  and  $s$  are positive.
- If the  $c$  term is positive and the  $b$  term is negative, then both  $r$  and  $s$  are negative.
- If the  $c$  term is negative and the  $b$  term is positive, then the factor that is positive will have the greater absolute value. That is, if  $|r| > |s|$ , then  $r$  is positive and  $s$  is negative.
- If the  $c$  term is negative and the  $b$  term is negative, then the factor that is negative will have the greater absolute value. That is, if  $|r| > |s|$ , then  $r$  is negative and  $s$  is positive.

After you have factored a number of trinomials in the form  $x^2 + bx + c$ , you may notice that the numbers you identify for  $r$  and  $s$  end up being included in the factored form of the trinomial. Have a look at the following chart, which reviews the three problems you have seen so far.

<b>Trinomial</b>	$x^2 + 7x + 10$	$x^2 + 5x + 6$	$x^2 + x - 12$
<b>r and s values</b>	$r = +5, s = +2$	$r = +2, s = +3$	$r = +4, s = -3$
<b>Factored form</b>	$(x + 5)(x + 2)$	$(x + 2)(x + 3)$	$(x + 4)(x - 3)$

Notice that in each of these examples, the  $r$  and  $s$  values are repeated in the factored form of the trinomial.

So what does this mean? It means that in trinomials of the form  $x^2 + bx + c$  (where the coefficient in front of  $x^2$  is 1), if you can identify the correct  $r$  and  $s$  values, you can effectively skip the grouping steps and go right to the factored form. You may want to stick with the grouping method until you are comfortable factoring, but this is a neat shortcut to know about!

Jess is trying to use the grouping method to factor the trinomial  $v^2 - 10v + 21$ . How should she rewrite the central  $b$  term,  $-10v$ ?

A)  $+7v + 3v$

B)  $-7v - 3v$

C)  $-7v + 3v$

D)  $+7v - 3v$

Show/Hide Answer

### Identifying Common Factors

Not all trinomials look like  $x^2 + 5x + 6$ , where the coefficient in front of the  $x^2$  term is 1. In these cases, your first step should be to look for common factors for the three terms.

Trinomial	Factor out Common Factor	Factored
$2x^2 + 10x + 12$	$2(x^2 + 5x + 6)$	$2(x + 2)(x + 3)$
$-5a^2 - 15a - 10$	$-5(a^2 + 3a + 2)$	$-5(a + 2)(a + 1)$
$c^3 - 8c^2 + 15c$	$c(c^2 - 8c + 15)$	$c(c - 5)(c - 3)$
$y^4 - 9y^3 - 10y^2$	$y^2(y^2 - 9y - 10)$	$y^2(y - 10)(y + 1)$

Notice that once you have identified and pulled out the common factor, you can factor the remaining trinomial as usual. This process is shown below.

Example

Problem	
<b>Factor <math>3x^3 - 3x^2 - 90x</math>.</b>	
$3(x^3 - x^2 - 30x)$	Since 3 is a common factor for the three terms, factor out the 3.
$3x(x^2 - x - 30)$	$x$ is also a common factor, so factor out $x$ .  Now you can factor the trinomial $x^2 - x - 30$ . To find $r$ and $s$ , identify two numbers whose product is $-30$ and whose sum is $-1$ .
$3x(x^2 - 6x + 5x - 30)$	The pair of factors is $-6$ and $5$ . So replace $-x$ with $-6x + 5x$ .
$3x[(x^2 - 6x) + (5x - 30)]$	Use grouping to consider the terms in pairs.
$3x[(x(x - 6) + 5(x - 6))]$	Factor $x$ out of the first group and factor $5$ out of the second group.
$3x(x - 6)(x + 5)$	Then factor out $x - 6$ .
<i>Answer</i>	$3x(x - 6)(x + 5)$

### Factoring Trinomials: $ax^2 + bx + c$

The general form of trinomials with a leading coefficient of  $a$  is  $ax^2 + bx + c$ . Sometimes the factor of  $a$  can be factored as you saw above; this happens when  $a$  can be factored out of all three terms. The remaining trinomial that still needs factoring will then be simpler, with the leading term only being an  $x^2$  term, instead of an  $ax^2$  term.

However, if the coefficients of all three terms of a trinomial don't have a common factor, then you will need to factor the trinomial with a coefficient of something other than 1.

#### Factoring Trinomials in the form $ax^2 + bx + c$

To factor a trinomial in the form  $ax^2 + bx + c$ , find two integers,  $r$  and  $s$ , whose sum is  $b$  and whose product is  $ac$ . Rewrite the trinomial as  $ax^2 + rx + sx + c$  and then use grouping and the distributive property to factor the polynomial.

This is almost the same as factoring trinomials in the form  $x^2 + bx + c$ , as in this form  $a = 1$ . Now you are looking for two factors whose product is  $a \cdot c$ , and whose sum is  $b$ .

Let's see how this strategy works by factoring  $6z^2 + 11z + 4$ .



In this trinomial,  $a = 6$ ,  $b = 11$ , and  $c = 4$ . According to the strategy, you need to find two factors,  $r$  and  $s$ , whose sum is  $b$  (11) and whose product is  $ac$  (or  $6 \cdot 4 = 24$ ). You can make a chart to organize the possible factor combinations. (Notice that this chart only has positive numbers. Since  $ac$  is positive and  $b$  is positive, you can be certain that the two factors you're looking for are also positive numbers.)

Factors whose product is 24	Sum of the factors
$1 \cdot 24 = 24$	$1 + 24 = 25$
$2 \cdot 12 = 24$	$2 + 12 = 14$
<b><math>3 \cdot 8 = 24</math></b>	<b><math>3 + 8 = 11</math></b>
$4 \cdot 6 = 24$	$4 + 6 = 10$

There is only one combination where the product is 24 and the sum is 11, and that is when  $r = 3$ , and  $s = 8$ . Let's use these values to factor the original trinomial.

Example	
Problem	Factor $6z^2 + 11z + 4$ .
	$6z^2 + 3z + 8z + 4$ Rewrite the middle term, $11z$ , as $3z + 8z$ (from the chart above.)
	$(6z^2 + 3z) + (8z + 4)$ Group pairs. Use grouping to consider the terms in pairs.
	$3z(2z + 1) + 4(2z + 1)$ Factor $3z$ out of the first group and $4$ out of the second group.
	$(2z + 1)(3z + 4)$ Factor out $(2z + 1)$ .
Answer	$(2z + 1)(3z + 4)$

Before going any further, it is worth mentioning that not all trinomials can be factored using integer pairs. Take the trinomial  $2z^2 + 35z + 7$ , for instance. Can you think of two integers whose sum is  $b$  (35) and whose product is  $ac$  ( $2 \cdot 7 = 14$ )? There are none! This type of trinomial, which cannot be factored using integers, is called a prime trinomial.

Factor $3x^2 + x - 2$ .
A) $(3x + 2)(x - 1)$
B) $(3x - 2)(x + 1)$
C) $(3x + 1)(x - 2)$

D)  $(3x - 1)(x + 2)$

Show/Hide Answer

A)  $(3x + 2)(x - 1)$

Incorrect. The product of  $(3x + 2)(x - 1)$  is  $3x^2 - x - 2$ ; look for two numbers whose product is  $-6$  and whose sum is  $+1$ . Then use those numbers to factor by grouping. The correct answer is  $(3x - 2)(x + 1)$ .

B)  $(3x - 2)(x + 1)$

Correct. The product of  $(3x - 2)(x + 1)$  is  $3x^2 + x - 2$ .

C)  $(3x + 1)(x - 2)$

Incorrect. The product of  $(3x + 1)(x - 2)$  is  $3x^2 - 5x - 2$ ; look for two numbers whose product is  $-6$  and whose sum is  $+1$ . Then use those numbers to factor by grouping. The correct answer is  $(3x - 2)(x + 1)$ .

D)  $(3x - 1)(x + 2)$

Incorrect. The product of  $(3x - 1)(x + 2)$  is  $3x^2 + 5x - 2$ ; look for two numbers whose product is  $-6$  and whose sum is  $+1$ . Then use those numbers to factor by grouping. The correct answer is  $(3x - 2)(x + 1)$ .

## Negative Terms

In some situations,  $a$  is negative, as in  $-4h^2 + 11h + 3$ . It often makes sense to factor out  $-1$  as the first step in factoring, as doing so will change the sign of  $ax^2$  from negative to positive, making the remaining trinomial easier to factor.

### Example

Problem

Factor  $-4h^2 + 11h + 3$

$-1(4h^2 - 11h - 3)$

Factor  $-1$  out of the trinomial. Notice that the signs of all three terms have changed.

To factor the trinomial, you need to figure out how to rewrite  $-11h$ . The product of  $rs = 4 \cdot -3 = -12$ , and the sum of  $rs = -11$ .

$-1(4h^2 - 12h + 1h - 3)$

$r \cdot s = -12$	$r + s = -11$
$-12 \cdot 1 = -12$	$-12 + 1 = -11$
$-6 \cdot 2 = -12$	$-6 + 2 = -4$
$-4 \cdot 3 = -12$	$-4 + 3 = -1$

Rewrite the middle term  $-11h$  as  $-12h + 1h$ .

Group terms.

$$-1[(4h^2 - 12h) + (1h - 3)]$$

$$-1[4h(h - 3) + 1(h - 3)]$$

$$-1[(h - 3)(4h + 1)]$$

$$-1(h - 3)(4h + 1)$$

Factor out  $4h$  from the first pair. The second group cannot be factored further, but you can write it as  $+1(h - 3)$  since  $+1(h - 3) = (h - 3)$ . This helps with factoring in the next step.

Factor out a common factor of  $(h - 3)$ . Notice you are left with  $(h - 3)(4h + 1)$ ; the  $+1$  comes from the term  $+1(h - 3)$  in the previous step.

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Note that the answer above can also be written as  $(-h + 3)(4h + 1)$  or  $(h - 3)(-4h - 1)$  if you multiply  $-1$  times one of the other factors.

### Summary

Trinomials in the form  $x^2 + bx + c$  can be factored by finding two integers,  $r$  and  $s$ , whose sum is  $b$  and whose product is  $c$ . Rewrite the trinomial as  $x^2 + rx + sx + c$  and then use grouping and the distributive property to factor the polynomial.

When a trinomial is in the form of  $ax^2 + bx + c$ , where  $a$  is a coefficient other than 1, look first for common factors for all three terms. Factor out the common factor first, then factor the remaining simpler trinomial. If the remaining trinomial is still of the form  $ax^2 + bx + c$ , find two integers,  $r$  and  $s$ , whose sum is  $b$  and whose product is  $ac$ . Then rewrite the trinomial as  $ax^2 + rx + sx + c$  and use grouping and the distributive property to factor the polynomial.

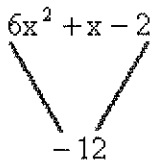
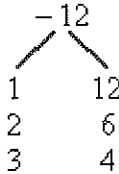
When  $ax^2$  is negative, you can factor  $-1$  out of the whole trinomial before continuing.



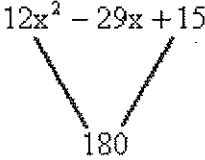
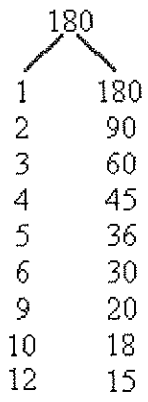
Here are the steps required for factoring a trinomial when the leading coefficient is not 1:

- Step 1:** Make sure that the trinomial is written in the correct order; the trinomial must be written in descending order from highest power to lowest power.
- Step 2:** Decide if the three terms have anything in common, called the greatest common factor or GCF. If so, factor out the GCF. Do not forget to include the GCF as part of your final answer.
- Step 3:** Multiply the leading coefficient and the constant, that is multiply the first and last numbers together.
- Step 4:** List all of the factors from Step 3 and decide which combination of numbers will combine to get the number next to x.
- Step 5:** After choosing the correct pair of numbers, you must give each number a sign so that when they are combined they will equal the number next to x and also multiply to equal the number found in Step 3.
- Step 6:** Rewrite the original problem with four terms by splitting the middle term into the two numbers chosen in step 5.
- Step 7:** Now that the problem is written with four terms, you can factor by grouping.

**Example 1** – Factor:  $6x^2 + x - 2$ 

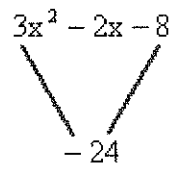
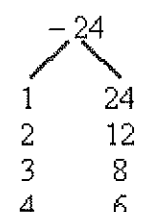
<p><b>Step 1:</b> Make sure that the trinomial is written in the correct order; the trinomial must be written in descending order from highest power to lowest power. In this case, the problem is in the correct order.</p>	$6x^2 + x - 2$
<p><b>Step 2:</b> Decide if the three terms have anything in common, called the greatest common factor or GCF. If so, factor out the GCF. Do not forget to include the GCF as part of your final answer. In this case, the three terms only have a 1 in common which is of no help.</p>	$6x^2 + x - 2$
<p><b>Step 3:</b> Multiply the leading coefficient and the constant, that is multiply the first and last numbers together. In this case, you should multiply 6 and <math>-2</math>.</p>	$6x^2 + x - 2$ 
<p><b>Step 4:</b> List all of the factors from Step 3 and decide which combination of numbers will combine to get the number next to <math>x</math>. In this case, the numbers 3 and 4 can combine to equal 1.</p>	$-12$ 
<p><b>Step 5:</b> After choosing the correct pair of numbers, you must give each number a sign so that when they are combined they will equal the number next to <math>x</math> and also multiply to equal the number found in Step 3. In this case, <math>-3</math> and <math>+4</math> combine to equal <math>+1</math> and <math>-3</math> times <math>+4</math> is <math>-12</math>.</p>	$-3 + 4 = +1$ <p style="text-align: center;">and</p> $(-3)(+4) = -12$
<p><b>Step 6:</b> Rewrite the original problem with four terms by splitting the middle term into the two numbers chosen in step 5.</p>	$6x^2 - 3x + 4x - 2$
<p><b>Step 7:</b> Now that the problem is written with four terms, you can factor by grouping.</p>	$3x(2x - 1) + 2(2x - 1)$ $(2x - 1)(3x + 2)$

**Example 2** – Factor:  $12x^2 - 29x + 15$ 

<p><b>Step 1:</b> Make sure that the trinomial is written in the correct order; the trinomial must be written in descending order from highest power to lowest power. In this case, the problem is in the correct order.</p>	$12x^2 - 29x + 15$
<p><b>Step 2:</b> Decide if the three terms have anything in common, called the greatest common factor or GCF. If so, factor out the GCF. Do not forget to include the GCF as part of your final answer. In this case, the three terms only have a 1 in common which is of no help.</p>	$12x^2 - 29x + 15$
<p><b>Step 3:</b> Multiply the leading coefficient and the constant, that is multiply the first and last numbers together. In this case, you should multiply 12 and 15.</p>	$12x^2 - 29x + 15$ 
<p><b>Step 4:</b> List all of the factors from Step 3 and decide which combination of numbers will combine to get the number next to x. In this case, the numbers 9 and 20 can combine to equal 29.</p>	
<p><b>Step 5:</b> After choosing the correct pair of numbers, you must give each number a sign so that when they are combined they will equal the number next to x and also multiply to equal the number found in Step 3. In this case, -9 and -20 combine to equal -29 and -9 times -20 is 180.</p>	$-9 - 20 = -29$ <p style="text-align: center;">and</p> $(-9)(-20) = 180$
<p><b>Step 6:</b> Rewrite the original problem with four terms by splitting the middle term into the two numbers chosen in step 5.</p>	$12x^2 - 9x - 20x + 15$
<p><b>Step 7:</b> Now that the problem is written with four terms, you can factor by grouping.</p>	$3x(4x - 3) - 5(4x - 3)$ $(4x - 3)(3x - 5)$

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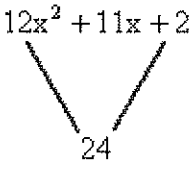
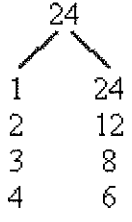
**Example 3** – Factor:  $6x^2 - 16 - 4x$ 

<p><b>Step 1:</b> Make sure that the trinomial is written in the correct order; the trinomial must be written in descending order from highest power to lowest power. In this case, the problem needs to be rewritten as:</p>	$6x^2 - 4x - 16$
<p><b>Step 2:</b> Decide if the three terms have anything in common, called the greatest common factor or GCF. If so, factor out the GCF. Do not forget to include the GCF as part of your final answer. In this case, the three terms have a 2 in common, which leaves:</p>	$2(3x^2 - 2x - 8)$
<p><b>Step 3:</b> Multiply the leading coefficient and the constant, that is multiply the first and last numbers together. In this case, you should multiply 3 and <math>-8</math>.</p>	$3x^2 - 2x - 8$ 
<p><b>Step 4:</b> List all of the factors from Step 3 and decide which combination of numbers will combine to get the number next to x. In this case, the numbers 4 and 6 can combine to equal 2.</p>	$-24$ 
<p><b>Step 5:</b> After choosing the correct pair of numbers, you must give each number a sign so that when they are combined they will equal the number next to x and also multiply to equal the number found in Step 3. In this case, <math>+4</math> and <math>-6</math> combine to equal <math>-2</math> and <math>+4</math> times <math>-6</math> is <math>-24</math>.</p>	$+4 - 6 = -2$ <p style="text-align: center;">and</p> $(+4)(-6) = -24$
<p><b>Step 6:</b> Rewrite the original problem with four terms by splitting the middle term into the two numbers chosen in step 5. Do not forget to include 2 (the GCF) as part of your answer.</p>	$2(3x^2 + 4x - 6x - 8)$
<p><b>Step 7:</b> Now that the problem is written with four terms, you can factor by grouping. Do not forget to include 2 (the GCF) as part of your final answer.</p>	$2(x(3x + 4) - 2(3x + 4))$ $2(3x + 4)(x - 2)$

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**Example 4** – Factor:  $36x^3 + 33x^2 + 6x$ 

<p><b>Step 1:</b> Make sure that the trinomial is written in the correct order; the trinomial must be written in descending order from highest power to lowest power. In this case, the problem is in the correct order.</p>	$36x^3 + 33x^2 + 6x$
<p><b>Step 2:</b> Decide if the three terms have anything in common, called the greatest common factor or GCF. If so, factor out the GCF. Do not forget to include the GCF as part of your final answer. In this case, the three terms have a <math>3x</math> in common, which leaves:</p>	$3x(12x^2 + 11x + 2)$
<p><b>Step 3:</b> Multiply the leading coefficient and the constant, that is multiply the first and last numbers together. In this case, you should multiply 12 and 2.</p>	$12x^2 + 11x + 2$ 
<p><b>Step 4:</b> List all of the factors from Step 3 and decide which combination of numbers will combine to get the number next to <math>x</math>. In this case, the numbers 3 and 8 can combine to equal 11.</p>	$24$ 
<p><b>Step 5:</b> After choosing the correct pair of numbers, you must give each number a sign so that when they are combined they will equal the number next to <math>x</math> and also multiply to equal the number found in Step 3. In this case, <math>+3</math> and <math>+8</math> combine to equal <math>+11</math> and <math>+3</math> times <math>+8</math> is 24.</p>	$+3 + 8 = +11$ <p style="text-align: center;">and</p> $(+3)(+8) = 24$
<p><b>Step 6:</b> Rewrite the original problem with four terms by splitting the middle term into the two numbers chosen in step 5. Do not forget to include <math>3x</math> (the GCF) as part of your answer.</p>	$3x(12x^2 + 3x + 8x + 2)$
<p><b>Step 7:</b> Now that the problem is written with four terms, you can factor by grouping. Do not forget to include 2 (the GCF) as part of your final answer.</p>	$3x(3x(4x + 1) + 2(4x + 1))$ $3x(4x + 1)(3x + 2)$

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**Note:** In each example above you could not use the shortcut because the leading coefficient was not a 1.

